V = burn velocity (m/s)
W_i = mass fraction of pyrolysis gas remaining in coal
$X^B_{O_2}$ = bulk oxygen mole fraction
Z' = fixed distance coordinate
Z = moving coordinate
Z_o = thickness of dry zone
α_v = reactivity factor
ϵ_o = porosity of wet coal
ϵ = porosity of dry coal
ϵ_d = initial porosity of dry coal at Z = 0
ρ^o_c = initial density of dry coal at Z = 0
ρ_o = density of wet coal
ρ_c = density of dry coal (variable)
μ_w = viscosity of water
μ = viscosity of gas
ν_{ij} = stoichiometric coefficient of ith species in jth reaction

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Hydrodynamic Model for Gas-Liquid Slug Flow in Vertical Tubes

A series of equations are developed based on the physical processes thought to take place during slug flow which are used to predict the hydrodynamic character of this complex flow pattern. New experimental results are reported which appear to validate the model.

R. C. FERNANDES,
R. SEMIAT and A. E. DUKLER
University of Houston
Houston, TX 77004

SCOPE

Gas-liquid slug flow in vertical tubes is a flow pattern commonly found in many industrial applications. It occurs in process equipment (vapor-liquid contactors or absorbers, vapor generators, thermosyphon reboilers and gas-liquid chemical reactors), in two-phase oil and gas pipelines, particularly in risers from subsea wells to platforms, as well as in oil, gas and geothermal wells.

The objective of this work was to develop a physically-based hydrodynamic model for this flow pattern and to obtain experimental data by which the model could be evaluated. The results should permit the calculation of holdup, pressure drop, characteristic velocities, frequency and backmixing characteristics for slug flow.

CONCLUSIONS AND SIGNIFICANCE

A model for two-phase slug flow has been constructed which can be used to predict many of the details of turbulent slug flow in vertical tubes including: average gas and liquid velocities in the slug and the Taylor bubbles separating successive slugs, the ratio of slug to bubble length as well as the average voids in the slug, at the Taylor bubble location and the average voids in the system. This information can then be used to calculate slugging frequency, pressure gradients and backmixing character of the flow.

Experimental measurements of certain of these variables were carried out in a 5.1 cm diameter vertical tube which was 11.1 m long using an air-water system. Satisfactory agreement between experimentally measured and predicted values was observed.

INTRODUCTION

When gas-liquid mixtures flow in a tube, the two phases distribute in a number of ways which characterize the spatial distribution of liquid and gas within the conduit. These are called flow patterns. A classification of flow patterns for the upward flow of gas-liquid mixtures in vertical pipes has been proposed by Hewitt and Hall-Taylor (1970), and by Taitel et al. (1980). Four dominant patterns are suggested: bubbly, slug, churn and annular flows. Slug flow is characterized by the presence of a series of large axisymmetric bullet-shaped gas bubbles, or Taylor bubbles, which occupy most of the cross section of the pipe and move essentially uniformly upward. Between the Taylor bubble and the pipe wall, liquid flows downward as a thin falling film. The successive Taylor bubbles are separated by regions of continuous liquid phase which bridge the channel and contain small gas bubbles. These structures are known as *liquid slugs*.

Slug flow occurs over a wide range of flow rate space and is frequently encountered in situations of industrial interest. Methods for predicting the operating conditions at which slug flow will exist were recently presented by Taitel et al. (1980) who developed physical models for flow pattern transitions during steady upward flow in a vertical tube. It is the intent of this paper to develop a model for the hydrodynamic characteristics of this type of flow.

ANALYSIS

The physical model developed below is based on equilibrium, isothermal, upward cocurrent gas-liquid flow in vertical pipes at low pressures. The two-phase flow will be considered as axisymmetric, one-dimensional and steady. While some difference in the dimensions and velocities of successive slugs and Taylor bubbles exist, these are observed to be small and the model developed below is completely deterministic. The results can be viewed as giving time averaged or ensemble average information. Under the conditions of fully developed flow, Taylor bubbles and liquid slugs rise steadily and follow one another in regular succession without any relative velocity between them (Nicklin et al., 1962; Stewart and Davidson, 1967; Jones and Zuber, 1975). Since the flow characteristics at any cross-sectional plane vary with time due to the intermittent nature of the slug pattern, a convenient modelling strategy is to consider a unit cell consisting of one Taylor bubble and its surrounding liquid film, plus one adjacent liquid slug. An idealized sketch of the slug unit is shown in Figure 1.

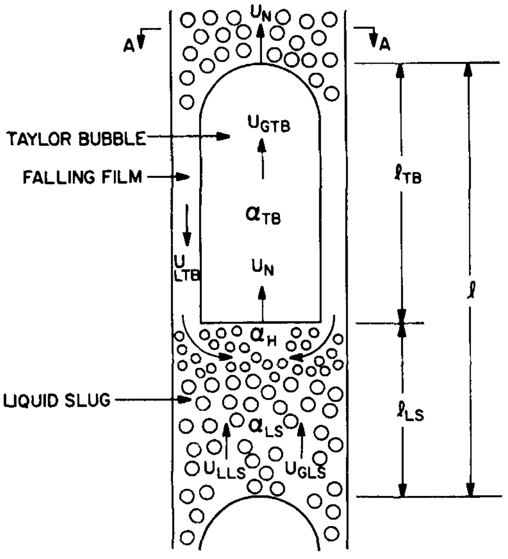

Figure 1. A slug unit.

Large Taylor bubbles which almost bridge the pipe translate steadily upward at a velocity U_N. These are long cylindrical voids having a nearly spherical nose and a flat tail. Their length, l_{TB}, remains essentially constant in the axial direction when expansion effects are small (e.g., for relatively short vertical columns and/or for high-pressure systems where the density of the gas will not change significantly as the bubble rises). Since, in general, the gas density and viscosity are much lower than the liquid density and viscosity, the pressure drop experienced by the gas in the Taylor bubble is very small. Thus, the interior of the bubble can be considered as a region of approximately constant pressure, the interfacial shear is negligible and the liquid film flows downward around the Taylor bubble as a free-falling film. The Taylor bubble is followed by a liquid slug containing significant amounts of small gas bubbles similar in size and motion to those observed in bubbly flow (Griffith and Wallis, 1961; Akagawa and Sakaguchi, 1966; Govier and Aziz, 1972; Taitel et al. 1980). The small bubbles distribute almost uniformly over the length of the liquid slug, except for a small region just behind the tail of the preceding Taylor bubble where the void concentration, α_H, is considerably higher than that in the bulk of the liquid slug, α_{LS}. This higher concentration is the result of entrainment of gas from the back of the Taylor bubble by the falling liquid film. This high void region can be easily observed in an experiment.

In stable slug flow the slug length, l_{LS}, remains constant as it propagates upward because the rate of addition of liquid from the falling film ahead of the slug is identically equal to that being shed as a film at the back of the same slug. In this sense the slug maintains its length in exactly the same way as observed by Dukler and Hubbard (1975) for horizontal slug flow.

As the liquid is shed from the back of a slug to form the falling film around the Taylor bubble, it is essentially free of the small bubbles seen in the slug. This bubble-free film results from the fact that the distributed bubbles in the slug are of a larger diameter than the thickness of the newly formed film. Thus, as they are carried downward with the liquid at the nose of the Taylor bubble, they coalesce with it. This falling film forms a mixing vortex at the front of the slug with a locally high concentration of gas. Figure 1 identifies the variables of interest.

where

l_{TB} = length of the Taylor bubble
l_{LS} = length of the liquid slug
l = length of the slug unit
α_{TB} = void fraction of the Taylor bubble
α_{LS} = void fraction of the liquid slug
α_H = void fraction of the region just behind the Taylor bubble

U_{GTB} = velocity of the gas in the Taylor bubble
U_{GLS} = velocity of the gas in the liquid slug
U_{LTB} = velocity of the liquid in the film around the Taylor bubble
U_{LLS} = velocity of the liquid in the liquid slug
U_N = velocity of translation of the Taylor bubble

All velocities and void fractions represent average values over their respective flow areas. It is the objective of this work to develop a model to predict these quantities as well as α_{SU}, the average volume void fraction of a slug unit.

Average Voids in a Slug Unit

The volume average void fraction over the slug unit, α_{SU}, is defined as

$$\alpha_{SU} \equiv \frac{V_G}{V_{SU}}, \tag{1}$$

where V_G represents the total volume of gas in the slug unit, and V_{SU} is the volume of the slug unit itself $= lA = (l_{TB} + l_{LS})A$.

$$V_G = V_{GTB} + V_{GLS}, \tag{2}$$

where V_{GTB} is the volume of gas in the Taylor bubble, V_{GLS} is the volume of gas in the liquid slug and A is the pipe flow area.

Assuming a uniform axial distribution of the gas within both the liquid slug and the Taylor bubble, Eq. 2 may be written as

$$V_G = l_{TB}\,A_{GTB} + l_{LS}\,A_{GLS},$$

where A_{GTB} represents the cross-sectional area of the cylindrical portion of the Taylor bubble, and A_{GLS} is the effective cross-sectional area occupied by the gas in the liquid slug. Hence,

$$\alpha_{SU} = \beta\alpha_{TB} + (1 - \beta)\alpha_{LS}, \tag{3}$$

where

$$\alpha_{TB} = \frac{A_{GTB}}{A}, \quad \alpha_{LS} = \frac{A_{GLS}}{A}, \quad \beta = \frac{l_{TB}}{l}. \tag{4}$$

According to Eq. 3, to determine the average gas holdup of the slug unit, α_{SU}, one needs to predict β, α_{TB} and α_{LS}.

Overall Mass Balances

The flow of the gas phase along a slug unit length is assumed to be incompressible, thus mass and volume balances are equivalent. Consider the flow of a slug unit through a fixed plane A-A located along the column at a position where fully developed slug flow exists, Figure 1. The time interval Δt_{TB} that the Taylor bubble takes to pass across section A-A is

$$\Delta t_{TB} = \frac{l_{TB}}{U_N}. \tag{5}$$

During this time the volume of gas carried upward by the Taylor bubble, crossing plane A-A, is

$$V_{GTB} = U_{GTB}A\alpha_{TB}\Delta t_{TB} = l_{TB}A\alpha_{TB}\frac{U_{GTB}}{U_N}. \tag{6}$$

The volume of gas, V_{GLS}, carried upward by the liquid slug that passes through section A-A during time interval, Δt_{LS}

$$\Delta t_{LS} = \frac{l_{LS}}{U_N} \tag{7}$$

is equal to

$$V_{GLS} = U_{GLS}A\alpha_{LS}\Delta t_{LS} = l_{LS}A\alpha_{LS}\frac{U_{GLS}}{U_N}. \tag{8}$$

During the time corresponding to the passage of one slug unit through plane A-A, $\Delta t = \Delta t_{TB} + \Delta t_{LS}$, the volume of gas entering the test section as feed is

$$Q_G(\Delta t_{TB} + \Delta t_{LS}) = U_{SG}A\left(\frac{l_{TB} + l_{LS}}{U_N}\right) \tag{9}$$

where Q_G is the volumetric rate of supply and U_{SG} is the superficial velocity of the gas. But according to the balance requirement this gas volume must be equal to the sum of gas volumes being carried by the Taylor bubble and liquid slug in a slug unit. Thus,

$$U_{SG}A\left(\frac{l}{U_N}\right) = l_{TB}A\alpha_{TB}\frac{U_{GTB}}{U_N} + l_{LS}A\alpha_{LS}\frac{U_{GLS}}{U_N}.$$

or

$$U_{SG} = \beta\alpha_{TB}U_{GTB} + (1 - \beta)\alpha_{LS}U_{GLS}. \tag{10}$$

The corresponding balance for the liquid phase flowing in a slug unit can be derived in the same manner as that for the gas. However, during the time interval Δt_{TB} there is a downward flow of the liquid (in the annular film around the Taylor bubble) passing through the fixed plane A-A. Thus,

$$U_{SL} = (1 - \beta)(1 - \alpha_{LS})U_{LLS} - \beta(1 - \alpha_{TB})U_{LTB} \tag{11}$$

where U_{SL} is the superficial velocity of the liquid.

Flow Relative to the Nose of Taylor Bubble

The Taylor bubble travels through the two-phase mixture of the liquid slug at a translation velocity, U_N, greater than that of either the gas or liquid phase which make up the slug. Thus, independent continuity relationships can be developed by considering the flow relative to the nose of the Taylor bubble. For the liquid phase one obtains

$$(U_N - U_{LLS})(1 - \alpha_{LS}) = (U_N + U_{LTB})(1 - \alpha_{TB}) \tag{12}$$

Equation 12 states that in a coordinate system which translates upward at a velocity, U_N, the rate of liquid flow approaching the nose from the slug is equal to that being drained in the film. The same procedure applied to the gas phase gives

$$(U_N - U_{GLS})\,\alpha_{LS} = (U_N - U_{GTB})\,\alpha_{TB} \tag{13}$$

Propagation Velocity of Taylor Bubbles

The rise velocity, U_R, of a single, nonexpanding Taylor bubble in stagnant liquid has been determined both theoretically and experimentally (Dumitrescu, 1943; Davies and Taylor, 1950; Nicklin et al., 1962) to be

$$U_R = 0.35(gD)^{1/2} \tag{14}$$

where D is the tube diameter. For the situation of a single Taylor bubble in a flowing turbulent liquid, Nicklin et al. (1962) suggested the following relationship

$$U_N = 1.2(U_{SG} + U_{SL}) + 0.35(gD)^{1/2} \tag{15}$$

where the coefficient 1.2 is perceived to be the ratio of the centerline to average velocity of the slug flow. A recent paper by Collins et al. (1978) provides strong theoretical support for this form of the equation.

Experiments to be described later, suggest a slight modification as follows

$$U_N = 1.29(U_{SG} + U_{SL}) + 0.35(gD)^{1/2} \tag{16}$$

The difference between the results given by Eq. 16 and that of Nicklin is attributed to the larger tube diameter of these experiments, some expansion effects which exist in the system and the possible interaction of successive Taylor bubbles.

Bubble Rise Velocity in the Slug

Except for the location immediately behind the Taylor bubble, the liquid and gas in the slug behave as in fully-developed bubbly flow which is continuous in the axial direction. Approximating the actual distributions which exist by cross-sectional average values, it is possible to write

$$U_{GLS} = U_{LLS} + U_O \tag{17}$$

where U_O is the rise velocity due to buoyancy. This velocity becomes independent of bubble size depending only on fluid properties for bubble sizes observed in two-phase flows. Zuber and Hench (1962) modified relationships presented by Harmathy (1960) for single bubbles to present an expression for the rise velocity of a bubble in the presence of a bubble swarm

$$U_O = 1.53 \left[\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right]^{1/4} (1 - \alpha)^{1/2} \tag{18}$$

Thus for a liquid slug

$$U_{GLS} = U_{LLS} + 1.53 \left[\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right]^{1/4} (1 - \alpha_{LS})^{1/2} \tag{19}$$

Falling Film

The annular film around the Taylor bubble is assumed to behave as a falling film without interfacial shear. The thickness and flow rate relationship in the film at the bottom of the Taylor bubble is assumed identical to that for a falling film which is created at the top surface of a vertical plate or cylinder. This can be expected to be a valid assumption if the entry length region for developing the velocity profile is less than the length of the film. These entry lengths are known to be of the order 100 times the mean film thickness. In this case the developing length ranges are of the order of 0.1 m. However, the length of film in a Taylor bubble ranges from 1 to 3 m. Thus, the premise seems fully acceptable. The thickness of falling films has been theoretically related to the flow rate and interfacial shear by Dukler (1959). Because of the complexity of the theory the results were presented graphically, which makes them inconvenient to use in the computer solution of an equation network. Brötz (1954) proposed an empirical correlation for the thickness of a free falling film from a study of films of water, pentadecane and a refrigerating oil in the turbulent flow regime, inside vertical tubes of various diameters. The film Reynolds number varied from 100 to 4,300. In dimensionless form Brötz correlation is

$$\delta_L \left(\frac{g}{\nu_L^2}\right)^{1/3} = \left(\frac{3\, Re_F^2}{590}\right)^{1/3} \tag{20}$$

where Re_F is the Reynolds number of the liquid film defined by

$$Re_F \equiv \frac{\delta_L U_{LTB}}{\nu_L} \tag{21}$$

and δ_L is the mean film thickness, ν_L is the kinematic viscosity of the liquid, and g is the gravitational acceleration. A comparison of Brötz equation with the theoretical predictions of Dukler shows excellent agreement (Drew et al., 1964). Thus, the use of this equation can simply be considered a fit to Dukler's theory, which was shown to be in agreement with data over a wide range of Reynolds numbers.

Substituting Definition 21 into Eq. 20 gives

$$U_{LTB}^2 = 196.7\, g\delta_L \tag{22}$$

The mean film thickness, δ_L, can be related to the area average void fraction of the Taylor bubble, α_{TB}

$$\delta_L = \frac{D}{2}(1 - \alpha_{TB}^{1/2}) \tag{23}$$

Incorporating Eq. 23 into Eq. 22 gives

$$U_{LTB} = 9.916[gD(1 - \alpha_{TB}^{1/2})]^{1/2} \tag{24}$$

There are now eight independent equations involving nine unknowns. An additional equation is needed to effect closure for the system.

Gas Flow Through Taylor Bubble

The Taylor bubble remains constant in length as it travels up the tube. Thus, the flux of gas into and out of the Taylor bubble must be equal. As the liquid in the slug flows around the nose of the Taylor bubble, it forms a film too thin to accommodate the bubbles carried with that liquid. These bubbles coalesce with the Taylor bubble forming a flow rate of gas entering the Taylor bubble at the top, Q_A, Figure 2. Photographs of slug flow indeed show that the lower portions of the liquid film are bubble-free.

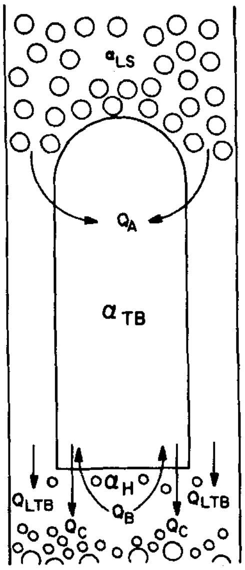

Figure 2. Gas flow around the Taylor bubble.

Two other paths for gas flow exist to make for the balance. At the bottom of the Taylor bubble the falling film entrains gas at a flow rate Q_C as it mixes with the liquid slug below, forming a region of high local gas concentration, α_H. Part of the gas returns to the Taylor bubble across its bottom plane, at a rate Q_B. The condition of no accumulation requires that

$$Q_A + Q_B = Q_C \tag{25}$$

Models for each of the terms in Eq. 25 are developed below.

The Taylor bubble overtakes the small bubbles in the liquid slug. Thus, the volumetric rate of gas capture is

$$Q_A = \frac{\pi}{4} D^2 \alpha_{LS}(U_N - U_{GLS}) \tag{26}$$

Figure 3 shows the process of gas entrainment into the slug following a Taylor bubble. The average velocity of the falling liquid film of thickness δ_L is U_{LTB}. The local velocity in the film varies from zero at the wall to $\hat{U}_{LS}$ at the surface. For the turbulent films existing under most conditions $\hat{U}_{LS} \simeq 1.15\, U_{LTB}$ (Portalski, 1964). The gas within the Taylor bubble has a velocity distribution $\hat{u}_G(y)$ varying from $-\hat{U}_{LS}$ to a value greater than U_N at the centerline. The front of the liquid slug following the Taylor bubble is rising at a velocity of U_N. All points in the gas where the local velocity is less than U_N will be overtaken by the liquid, the gas will be captured and entrained into the slug. Define δ_G as the radial distance from the interface to the position in the gas phase where the local velocity, $\hat{u}_G = U_N$. Then

$$Q_C = 2\pi \int_{\delta_L}^{(\delta_L + \delta_G)} \left(\frac{D}{2} - y\right) \hat{u}_r dy \tag{27}$$

where $\hat{u}_r = U_N - \hat{u}_G$ and y is the coordinate measured from the wall. The auxiliary conditions are:

$$y = \delta_L \qquad \hat{u}_r = 1.15\, U_{LTB} + U_N \tag{28}$$

$$y = \delta_L + \delta_G \qquad \hat{u}_r = 0 \tag{29}$$

Define the variables

$$\hat{u}' = \hat{u}_G + 1.15\, U_{LTB} = -\hat{u}_r + 1.15\, U_{LTB} + U_N \tag{30}$$

and

$$y' = y - \delta_L \tag{31}$$

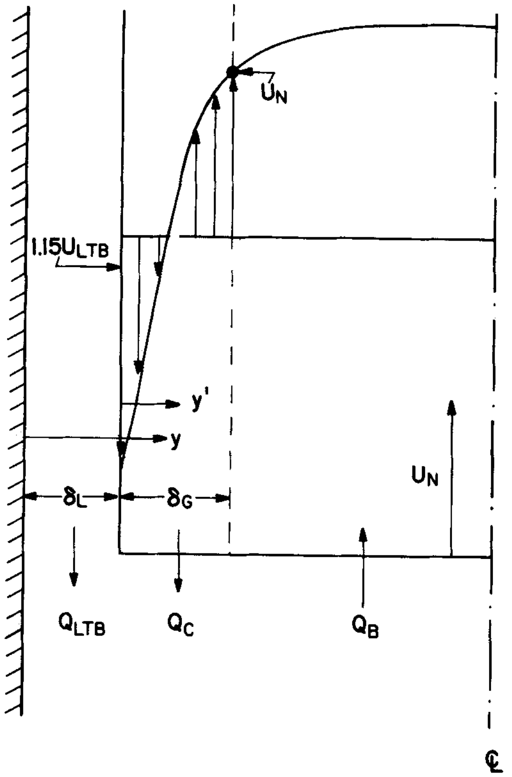

Figure 3. The process of gas entrainment at the top of a liquid slug.

the auxiliary conditions become

$$y' = 0 \qquad \hat{u}' = 0 \tag{32}$$

$$y' = \delta_G \qquad \hat{u}' = 1.15\, U_{LTB} + U_N \tag{33}$$

Substituting in Eq. 27 gives

$$Q_C = \pi \delta_G (1.15\, U_{LTB} + U_N)(D - 2\delta_L - \delta_G) - 2\pi \int_0^{\delta_G} \left(\frac{D}{2} - \delta_L - y'\right) \hat{u}' dy' \tag{34}$$

In most practical cases the gas flow is turbulent and it is possible to use the universal velocity profile for turbulent flow to evaluate the integral in Eq. 34. Introducing the dimensionless variables characteristic of the universal velocity distribution

$$u^+ = \frac{\hat{u}'}{U_*} \qquad y^+ = \frac{y' U_*}{\nu_G} \tag{35}$$

$$\delta_G^+ = \frac{\delta_G U_*}{\nu_G} \qquad \frac{D^+}{2} = \left(\frac{D}{2} - \delta_L\right) \frac{U_*}{\nu_G} \tag{36}$$

where ν_G is the kinematic viscosity of the gas and U_* is a friction velocity. Substituting in Eq. 34 gives

$$Q_C = \pi \delta_G (1.15\, U_{LTB} + U_N)(D - 2\delta_L - \delta_G) - \frac{2\pi \nu_G^2}{U_*} \int_0^{\delta_G^+} \left(\frac{D^+}{2} - y^+\right) u^+ dy^+ \tag{37}$$

To compute Q_C, additional relations for δ_G and U_* are required.

The thickness of the gas layer, δ_G, can be computed from the condition that, at $y' = \delta_G$,

$$u^+(@y^+ = \delta_G^+) = \frac{1.15\, U_{LTB} + U_N}{U_*} \tag{38}$$

The same universal velocity distribution can be integrated from the interface to the center of the pipe to calculate the average volumetric flow rate of the gas being carried upward in the Taylor bubble,

$$Q_{GTB} = 2\pi \int_{\delta_L}^{D/2} \left(\frac{D}{2} - y\right) \hat{u} dy = \pi \left(\frac{D}{2} - \delta_L\right)^2 U_{GTB} \tag{39}$$

Making the transformations as previously to a gas velocity, $\hat{u}'$, measured relative to the interface and then to dimensionless coordinates, yields

$$\int_0^{D^+/2} \left(\frac{D^+}{2} - y^+\right) u^+ dy^+ = \frac{\left(\frac{D}{2} - \delta_L\right)^2}{2\nu_G^2} \times U_* (1.15\, U_{LTB} + U_{GTB}) \tag{40}$$

Equations 38 and 40 provide two equations for the two new unknowns δ_G and U_* introduced in Eq. 37. The integral in Eqs. 37 and 40 can be evaluated using the convenient three equation form for the distribution

$$\begin{aligned} u^+ &= y^+ && 0 \le y^+ < 5 \\ u^+ &= 5.0\, ln y^+ - 3.05 && 5 \le y^+ \le 30 \\ u^+ &= 2.5\, ln y^+ + 5.5 && 30 \le y^+ \end{aligned} \tag{41}$$

Designate I as the integral

$$I = \int_0^{M^+} \left(\frac{D^+}{2} - y^+\right) u^+ dy^+ \tag{42}$$

The integration gives, for the usual case of $M^+ \ge 30$,

$$I = 573.202 - 63.895 \left(\frac{D^+}{2}\right) + [2.5\, m^+ ln(m^+) + 3.0 m^+] \left(\frac{D^+}{2}\right) - 1.25\, (m^+)^2\, ln(m^+) - 2.125\, (m^+)^2 \tag{43}$$

A model for Q_B can be developed by considering the high void region formed behind the Taylor bubble as the falling film and entrained gas merge with the liquid slug below. The high local voids, α_H, which exists at that point can be estimated from

$$\alpha_H = \frac{Q_C}{Q_C + Q_{LTB}} \tag{44}$$

where Q_C is determined from Eq. 34 and Q_{LTB}, the volumetric flow rate of the falling film, can be calculated as follows:

$$Q_{LTB} = \frac{\pi}{4} D^2 (1 - \alpha_{TB})(U_{LTB} + U_N) \tag{45}$$

This region at the front of the slug is a highly turbulent one as a result of the merging of a wall jet consisting of a liquid film interacting with the liquid rising in the slug. As a result the turbulent intensity is enhanced and these high axial velocity fluctuations result in transport of bubbles from the high voids region back toward the interface between the front of the slug and the back of the Taylor bubble creating a flow, Q_B. Visual observations clearly show such a migration and reentry of bubbles from the front of the slug into the back of the Taylor bubble.

From this argument one can write

$$Q_B = \frac{\pi}{4} [D - 2(\delta_L + \delta_G)]^2\, \alpha_H U_{RMS} \tag{46}$$

where U_{RMS} is the intensity of turbulence.

The falling film can be considered as a wall jet entering a stagnant pool of liquid at a velocity $(U_{LTB} + U_N)$. Bradshaw (1971), Hinze (1975), and Schlichting (1979) report on the turbulent intensity associated with the mixing of free jets as they flow into stationary pools of the same fluid. The ratio of intensities at the edge of the jets to the maximum entering velocities are shown to range between 0.2 to 0.3. Although this is not a free jet, the work of Schlichting (1979) and others suggests that the outer or free boundary of a wall jet behaves like that of a free jet. Thus, for purposes of this model we assume

$$\frac{U_{RMS}}{1.15\, U_{LTB} + U_N} = 0.25 \tag{47}$$

Substituting Eq. 47 into Eq. 46 gives

$$Q_B = \frac{\pi}{16} [D - 2(\delta_L + \delta_G)]^2 \alpha_H [1.15\, U_{LTB} + U_N] \tag{48}$$

The model for gas flow through the Taylor bubble presented here appears to be supported by the studies of Harrison et al. (1961),

TABLE I
SUMMARY OF EQUATION NETWORK

	EQUATION NO.
Average Void Fraction over a Slug Unit	
$\alpha_{SU} = \beta\alpha_{TB} + (1-\beta)\alpha_{LS}$	(3)
Mass Balance on Gas Flow	
$U_{SG} = \beta\alpha_{TB}U_{GTB} + (1-\beta)\alpha_{LS}U_{GLS}$	(10)
Mass Balance on Liquid Flow	
$U_{SL} = (1-\beta)(1-\alpha_{LS})U_{LLS} - \beta(1-\alpha_{TB})U_{LTB}$	(11)
Gas Flow Relative to Nose of Taylor Bubble	
$(U_N - U_{GLS})\alpha_{LS} = (U_N - U_{GTB})\alpha_{TB}$	(13)
Liquid Flow Relative to Nose of Taylor Bubble	
$(U_N - U_{LLS})(1-\alpha_{LS}) = (U_N + U_{LTB})(1-\alpha_{TB})$	(12)
Rise Velocity of Taylor Bubble	
$U_N = 0.35\,(gD)^{1/2} + 1.29(U_{SG} + U_{SL})$	(16)
Rise Velocity of Small Bubbles in the Liquid Slug	
$U_{GLS} = U_{LLS} + 1.53\left[\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right]^{1/4}(1-\alpha_{LS})^{1/2}$	(19)
Film Thickness/Flow Rate Relation for Falling Film	
$U_{LTB} = 9.916\left[gD\left(1-\alpha_{TB}^{1/2}\right)\right]^{1/2}$	(24)
Relation between Liquid Film Thickness and Voids	
$\delta_L = \frac{D}{2}\left(1-\alpha_{TB}^{1/2}\right)$	(23)

Filla et al. (1976), and Davidson et al. (1979). These investigators observed the presence of a toroidal vortex in the lower part of the Taylor bubble which is exactly what would be expected if gas is released upward into the bubble in its central region (Q_B) but changes direction and is entrained downward at its periphery (Q_C).

Summary of Equation Network

The network of equations available to solve this problem is summarized in Table 1. Note that there exist 17 independent equations. The unknowns are as follows:

$$\alpha_{SU},\ \alpha_{TB},\ \alpha_{LS},\ \alpha_H,\ \beta$$
$$U_{GTB},\ U_{GLS},\ U_{LLS},\ U_{LTB},\ U_N,\ U_*$$
$$\delta_L,\ \delta_G$$
$$Q_A,\ Q_B,\ Q_C,\ Q_{LTB}$$

which total 17. The equation network is thus sufficient to provide a closed-form solution. Fernandes (1981) has provided a straightforward method of solution using the bisection method and provides a program in Fortran language for that purpose.

Some Derived Quantities of Interest

The solution of the equation set and calculation of the hydrodynamic variables permits the determination of a number of operating quantities of interest. For example:

Slug Frequency:

$$\nu_S = \frac{U_N}{l} = \frac{U_N(1-\beta)}{l_{LS}} \tag{49}$$

Taitel et al. (1980) have speculated on theoretical grounds that stable liquid slugs have a length $l_{LS} \simeq 20\,D$. Photographic measurements made in this study generally confirm that idea. Thus, the slugging frequency can be calculated once β is found from the solution network discussed above.

$$\nu_S = \frac{[1.29(U_{SG} + U_{SL}) + 0.35\sqrt{gD}](1-\beta)}{20\,D} \tag{50}$$

Pressure Drop in the Slug.

In addition to a hydrostatic gradient in the slug, acceleration and frictional gradients can be expected. For example, the liquid film falling around the Taylor bubble must be reversed in direction and accelerated to U_{LLS}. The pressure gradient associated with this acceleration is estimated to be

$$\Delta P_A = \frac{\rho_L Q_{LTB}}{A}(U_{LTB} + U_{LLS})$$
$$= \rho_L U_{LTB}(1-\alpha_{TB})(U_{LTB} + U_{LLS}) \tag{51}$$

The frictional gradient can be calculated using the similarity

Auxiliary Condition at the Edge of the Gas Layer:
Match of Gas Velocity Distributions

$$U^+\left(@y^+ = \frac{\delta_G U_*}{\nu_G}\right) = \frac{1.15\,U_{LTB} + U_N}{U_*} \tag{38}$$

Flow Rate of Gas in the Taylor Bubble

$$\int_0^{D^+/2}\left(\frac{D^+}{2} - y^+\right)u^+ dy^+ = \frac{\left(\frac{D}{2} - \delta_L\right)^2}{2\nu_G^2}\,U_*(1.15\,U_{LTB} + U_{GTB}) \tag{40}$$

Gas Flow into Taylor Bubble at top

$$Q_A = \frac{\pi}{4}D^2\alpha_{LS}(U_N - U_{GLS}) \tag{26}$$

Gas Flow into Taylor Bubble at Bottom

$$Q_B = \frac{\pi}{4}\left[D - 2(\delta_L + \delta_G)\right]^2\alpha_H\left[0.25(1.15U_{LTB} + U_N)\right] \tag{48}$$

Gas Entrained from Taylor Bubble into Slug

$$Q_C = \pi\delta_G(1.15\,U_{LTB} + U_N)(D - 2\delta_L - \delta_G) - \frac{2\pi\nu_G^2}{U_*}\int_0^{\delta_G^+}\left(\frac{D^+}{2} - y^+\right)u^+ dy^+ \tag{37}$$

Liquid Film Flow Rate

$$Q_{LTB} = \frac{\pi D^2}{4}(1 - \alpha_{TB})(U_{LTB} + U_N) \tag{45}$$

Mixing Equation at top of Slug

$$\alpha_H = \frac{Q_C}{Q_C + Q_{LTB}} \tag{44}$$

Balance on Gas Flow through Taylor Bubble

$$Q_A + Q_B = Q_C \tag{25}$$

equations for distributed flow with slip as discussed by Dukler et al. (1964).

$$\Delta P_f = \frac{2 f_{TP}\rho_{TP}U_{LLS}^2}{D}\cdot l_{LS} = 40 f_{TP}\rho_{TP}U_{LLS}^2 \tag{52}$$

where f_{TP} and ρ_{TP} can be determined once α_{LS}, U_{LLS} and U_{GLS} are calculated from the model.

EXPERIMENTS

Experiments for fully developed turbulent slug flow were conducted with air-water mixtures flowing in a vertical smooth Plexiglas pipe 0.05074 m i.d. and 11.1 m total length. Air and water were mixed together in an injection nozzle; water flowed axially, while air was injected radially through a horizontal row of 80 cylindrical holes of diameter equal to 0.0016 m and 0.005 m long.

The range of conditions covered in these experiments is shown in Figure 4, where the points represent conditions of the experimental runs and the curves represent the theoretical transition boundaries (Taitel et al., 1980).

Volume-average void fraction measurements over a wide range of liquid and gas rates were accomplished by a special quick closing valve system, which consisted of two identical valve/actuator sets. The valves utilized were full ported ball valves so that when they were fully open the flow was completely unobstructed. The spring-diaphragm actuators were specially designed to ensure a fast closing action of the ball valves. The total time interval required to move the ball from a fully-open to a fully-closed position was determined as 0.053 s. The actuators were connected to a common on-off electrical switch to ensure a simultaneous closure of the valves.

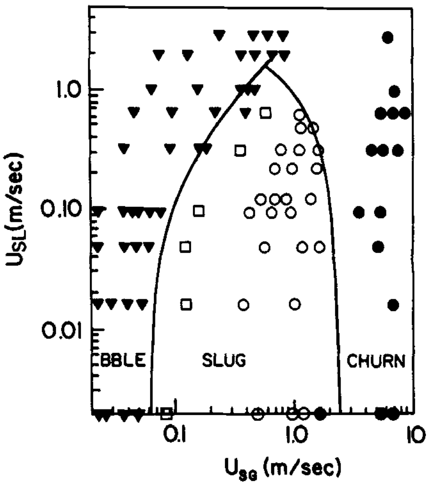

Figure 4. Range of experimental conditions: (open circled indicate slug flow was observed. Squares indicate transition of bubble to slug pattern.)

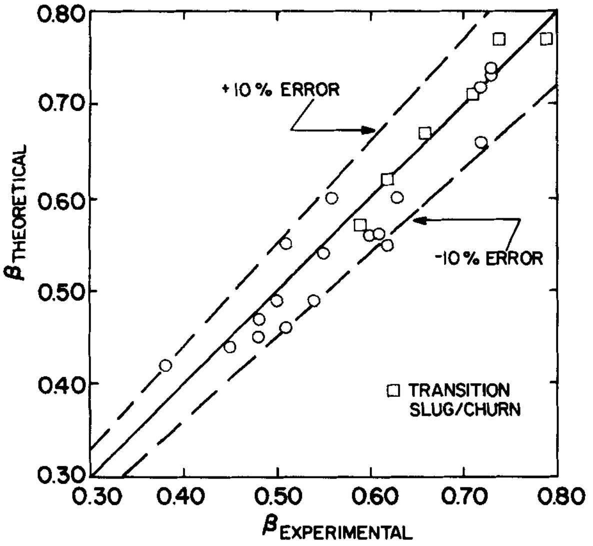

Figure 5. Comparison of theoretical and measured values of β.

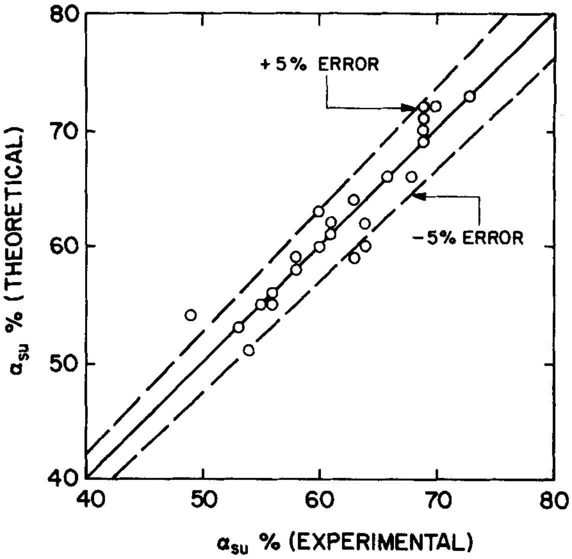

Figure 7. Comparison of theoretical and measured values of α_{SU}.

The length of the measuring station was made short—equal to 0.346 m—so that it could entrap the central portion of liquid slugs or of Taylor bubbles, while the distance between the injection nozzle and the measuring station was made long—equal to 130 pipe diameters—to ensure fully-developed slug flow within the measuring station.

The relative experimental uncertainties in the measured void fractions were less than 1%. The void fraction of liquid slugs and Taylor bubbles measured by the fast closing valves in the present work was assumed equal to the respective area average void fractions, α_{LS} and α_{TB}.

Velocities and lengths of Taylor bubbles and liquid slugs were measured by means of a photographic technique which utilized a 16 mm movie camera at a speed of 16 frames/second. The photographs were taken with the camera fixed with respect to the vertical column. From these measurements the ratio between the length of the Taylor bubble and the length of the slug unit, β, could be calculated for each gas-liquid flow rate pair. With the measured values of α_{TB}, α_{LS}, and β, the volume-average void fraction of a slug unit, α_{SU}, could be determined. The maximum relative experimental uncertainty in the length ratio β was 9.2%, while for α_{SU} it was equal to 4.1%.

Owing to the stochastic nature of gas-liquid slug flows, repeated measurements of void fractions, velocities and lengths of Taylor bubbles and liquid slugs were taken for each given pair of superficial velocities, U_{SG}, U_{SL}. From each set of measurements a sample mean and a sample standard deviation were computed. The number of measurements ranged from 10 to 20 times. Tabulated data and a detailed discussion of the experimental study may be found in Fernandes (1981).

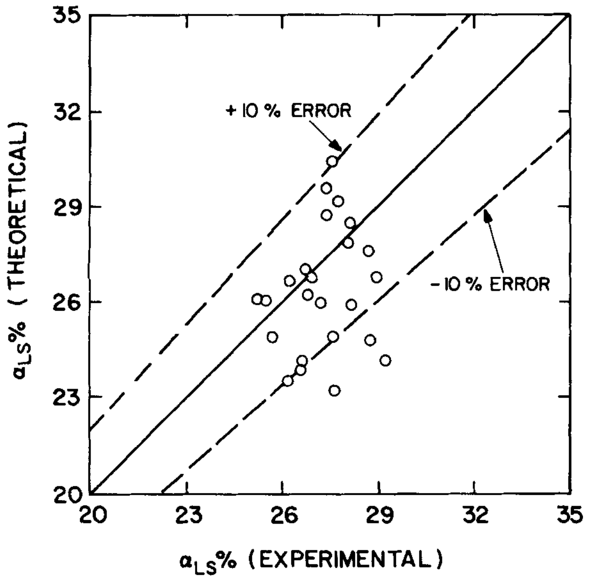

Figure 6. Comparison of theoretical and measured values of α_{LS}.

COMPARISON OF THEORY WITH EXPERIMENT

Theoretically computed values were obtained from the slug-flow model for each experimental condition. The predictions for U_N, U_{GTB}, U_{GLS}, U_{LTB}, U_{LLS}, U_*, δ_G, δ_L, β, α_H, α_{TB}, α_{LS} and α_{SU} are presented in tabulated form by Fernandes (1981). The experimental and theoretical results for β, α_{LS} and α_{SU}, which were measured, are presented and compared graphically in Figures 5 through 7. In all cases the agreement is quite satisfactory. The measured values of α_{TB} fell in the narrow range of 83.2 to 87.2%, while the theoretical values ranged from 85.8 to 88.1%. Average relative error in predicting α_{TB} defines as

$$\frac{|(\alpha_{TB})_{\text{exp}} - (\alpha_{TB})_{\text{theory}}|}{(\alpha_{TB})_{\text{exp}}}$$

was 2.5% with the maximum value being 5.5%. In general, the theory predicted high.

Simplified Approach to the Model

The first eight equations of Table 1 contain nine unknowns, β, α_{TB}, α_{LS}, α_{SU}, U_N, U_{GTB}, U_{GLS}, U_{LLS}, and U_{LTB}. One approach to a partial solution is to assume that α_{LS} takes the value which exists at the transition from bubbly to slug flow. Taitel et al. (1980) have suggested this transition value to be 0.25. Thus, setting $\alpha_{LS} = 0.25$ provides for closure. The more limited results for the variables which can be calculated from this abbreviated model are in reasonable agreement with the predictions of the more complete model, although the information developed is much more limited in character.

NOTATION

A = cross-sectional area of the pipe
A_{GLS} = cross-sectional area occupied by gas in liquid slug
A_{GTB} = cross-sectional area of cylindrical portion of Taylor bubble
D = internal pipe diameter
D^+ = dimensionless diameter of cylindrical region of Taylor bubble, defined by Eq. 36
f_{TP} = two-phase friction factor for liquid slug
g = acceleration of gravity
l = length of slug unit
l_{LS} = length of liquid slug
l_{TB} = length of Taylor bubble
Q_A = volumetric gas flow rate entering Taylor bubble from above (relative to the Taylor bubble), defined by Eq.

26

Q_B = volumetric gas flow rate entering Taylor bubble from below (relative to the Taylor bubble), defined by Eq. 46

Q_C = volumetric flow rate of gas leaving Taylor bubble in gas layer (relative to the Taylor bubble), defined by Eq. 37

Q_G = volumetric gas flow rate to column

Q_{GTB} = average volumetric flow rate of gas being transported upward in the Taylor bubble (relative to pipe wall), defined by Eq. 39

Q_{LTB} = volumetric flow rate of liquid being transported downward around the Taylor bubble (relative to the Taylor bubble), defined by Eq. 45

Re_F = Reynolds number of falling film, Eq. 21

U_* = friction velocity

U_{GLS} = area average velocity of gas in liquid slug

U_{GTB} = area average velocity of gas in Taylor bubble

U_{LLS} = area average velocity of liquid in liquid slug

$\hat{U}_{LS}$ = local velocity of liquid at free-surface of film

U_{LTB} = area average velocity of liquid in film around the Taylor bubble

U_N = velocity of translation of Taylor bubble

U_o = relative velocity between gas and liquid phase in bubbly flow

U_R = terminal rise velocity of a single Taylor bubble in a stagnant liquid

U_{RMS} = root mean square value of axial velocity fluctuations in near wake of Taylor bubble

U_{SG} = superficial velocity of gas

U_{SL} = superficial velocity of liquid

u^+ = dimensionless relative velocity, defined by Eq. 35

$\hat{u}'$ = local velocity of the gas within Taylor bubble relative to film surface velocity, defined by Eq. 30

$\hat{u}_G$ = local velocity of gas in Taylor bubble

$\hat{u}_r$ = local velocity of gas in gas layer relative to Taylor bubble rise velocity

V_{GLS} = volume of gas in liquid slug

V_{GTB} = volume of gas in Taylor bubble

y = distance measured from pipe wall

y^+ = dimensionless distance measured from liquid film surface in Taylor bubble, defined by Eq. 35

y' = distance from liquid film surface in Taylor bubble, defined by Eq. 31

Greek Letters

α_H = area average void fraction in liquid slug region adjacent to tail of Taylor bubble

α_{LS} = area average void fraction in the bulk of liquid slug

α_{SU} = volume average void fraction of a slug unit

α_{TB} = area average void fraction in cylindrical portion of Taylor bubble

β = ratio between length of Taylor bubble and length of slug unit, defined by Eq. 4

δ_G = mean thickness of gas layer within Taylor bubble

δ_G^+ = dimensionless mean thickness of gas layer, defined by Eq. 36

δ_L = mean thickness of liquid film around Taylor bubble

ν_L = kinematic viscosity of liquid

ν_s = slug frequency

ρ_k = density of phase k; $k = G, L$

ρ_{TP} = two-phase density in liquid slug

σ = surface tension

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